Qualitative Chain Graphs and their Use in Medicine

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Motivation: modelling PGMs in medicine

- Underlying **physiological processes**: dynamic (feedback) systems
  - homeostasis is ensured (**equilibrium** state)
- Disturbances may lead to suboptimal equilibria (**disease**)
- **Treatments** may affect the ‘setpoint’ of these systems
- Example:

![Diagram](image.png)
Chain graph as equilibrium of causal feedback

Example LWF chain graph (Lauritzen and Richardson)

The distribution of the chain graph model:

```
  a → c
    |
  b → d
```

represents the equilibrium of a process represented by an infinite DAG:

```
a  c
  |   |   |
  v   v   v
  c0 → c1 → ci → ci+1 → di → di+1
  d0 → d1 → di
  b
```
The example is modelled as a chain graph:

\[ \Pr(Ob, Th, LD, DM, Ch, Gl) \propto \Pr(Ch \mid LD) \cdot \Pr(Gl \mid DM) \cdot \varphi_1(LD, DM, Ob) \cdot \varphi_2(Ob, Th, DM) \cdot \Pr(Ob) \cdot \Pr(Th) \]

\( \varphi_i \) are black-box parameters.
Problem: it can be difficult to exploit human knowledge in assessing chain graph parameters

Goal: qualitative abstraction of chain graphs

Approach: qualitative relationships based on qualitative probabilistic networks

Qualitative and quantitative knowledge is combined

Use such qualitative knowledge for making decisions
Qualitative probabilistic networks (QPNs)

- Qualitative abstractions of Bayesian networks

- Instead of a conditional probability $P(B \mid \pi(B))$, qualitative properties of the conditional probability are associated to each node $B$
  - Qualitative influences $S^\delta(A, B)$: the effect of a cause $A$ on $B$ (all other things being equal)
  - Qualitative synergies: interaction of two causes on the effect
    - Additive synergy $Y^\delta(\{A_1, A_2\}, B)$
    - Product synergy $X^\delta(\{A_1, A_2\}, b)$

- Probabilistic relationships have signs $\delta \in \{+, -, 0, ?\}$
Qualitative influences in chain graphs

- In QPNs: the influence of $A$ on $B$ is $\delta$ if
  
  $$\delta = \text{sign}(P(b \mid a, x) - P(b \mid \bar{a}, x))$$

  for all configuration $x$ of other parents of $B$; $\delta = ?$ otherwise

- Probabilistic chain graphs: neighbours need to be considered

Causal definition of influence

The influence of $A$ on $B$ in a context $c \in V - AB$ is

$$P(b \parallel a, c) - P(b \parallel \bar{a}, c)$$

where $P(X \parallel Y = y)$ denotes the probability of $X$ after the intervention $Y = y$
Chain graph influence

Given two nodes $A$ and $B$ and a context $c$, then the influence of $A$ on $B$ in context $c$ equals:

$$P(b \mid a, z) - P(b \mid \overline{a}, z)$$

where $c = z \cup x$, $Z = \text{bd}(B) - A$, and $X = V - ZAB$.

The influence of $Ob$ on $DM$ is:

$$P(dm \mid ob, Th, LD) - P(dm \mid \overline{ob}, Th, LD)$$

in any context $\{Th, LD, Ch, Gi\}$.
QPN concepts can then be defined for qualitative chain graphs:

**Influences**

For example: \( S^+(A, B) \) if \( A \in \text{bd}(B) \) and

\[
P(b \mid a, \text{bd}(B) - A) \geq P(b \mid \bar{a}, \text{bd}(B) - A)
\]

**Synergies**

For example: \( Y^+(\{A_1, A_2\}, B) \) if \( A_1, A_2 \in \text{bd}(B) \), \( Z = \text{bd}(B) - A_1A_2 \), and

\[
P(b \mid a_1, a_2, Z) - P(b \mid \bar{a}_1, a_2, Z) \\
\geq P(b \mid a_1, \bar{a}_2, Z) - P(b \mid \bar{a}_1, \bar{a}_2, Z)
\]

\( \Rightarrow \) Other QPN concepts can be defined similarly
It holds that qualitative signs of chain graphs are symmetric, i.e., suppose \((A, B) \in E\), then \(P(b \mid a, X) - P(b \mid \bar{a}, X) \geq 0\) if and only if \(P(a \mid b, Y) - P(a \mid \bar{b}, Y) \geq 0\), where \(X = \text{bd}(B) - A\) and \(Y = \text{bd}(A) - B\).
Reasoning with qualitative chain graphs

- In QPNs, conclusions are derived based on the signs (arc reversal or sign propagation)

- Alternative approach is to look upon qualitative influences/synergies as constraints (Druzdzel and van der Gaag, 1995)
  1. Sample parameters consistent with constraints
  2. Perform inference in each network
  3. Derive confidence intervals for marginals

- Can combine qualitative and quantitative information

- Locality of constraints can be exploited during sampling (come to the poster..)
Example

\[ P(Ob) = 0.3 \quad P(Th) = 0.5 \]

\[ S^+(Ob, DM) \]
\[ S^-(Th, DM) \]
\[ S^+(LD, DM) \]
\[ Y^+(\{Ob, Th\}, DM) \]

\[ P(Ch | LD) = 0.8 \]
\[ P(Ch | \overline{LD}) = 0.3 \]

\[ P(Ch | Th) (82\% > P(Ch)) \]
\[ P(Ch | Th, Ob) (91\% > P(Ch)) \]
Conclusions and future work

Conclusions:

▶ Feedback systems relevant in many domains (medicine, economics, embedded systems, etc)
▶ Qualitative chain graph models allow combining qualitative and quantitative information to model such systems
▶ While not precise, can be used for decision making

Future work:

▶ Application to multiple feedback systems (diabetes, cardiovascular domains)
▶ Extending the theory and efficiency of reasoning