Appendix A. Supplementary Material

Consider a closed and convex set $C$ in $\mathbb{R}^d$. Let $d$ denote also the dimension of $C$. The vertices of $C$ are assumed to be finite and denoted as $e(C)$. A hyperplane $H$ in $\mathbb{R}^d$ can be parametrized by a pair $(v, w)$, with $v \in \mathbb{R}^d$ and $w \in \mathbb{R}$ as follows:

$$H_{v,w} := \{ x \in \mathbb{R}^d : v \cdot x = w \}. \quad (1)$$

The segment $S$ connecting points $a, b \in \mathbb{R}^d$ is instead:

$$S_{a,b} := \{ x \in \mathbb{R}^d : \lambda a + (1 - \lambda)b, 0 \leq \lambda \leq 1 \}. \quad (2)$$

**Definition 1** $H_{v,w}$ is a supporting hyperplane for $C$ passing through $x^* \in C$ if and only if $x^* \in P_{v,w}$ and $v \cdot x \leq v \cdot x^*$ for each $x \in C$.

**Definition 2** A point $x^*$ belongs to the boundary $b(C)$ of the convex set $C$ if and only if there is at least a supporting hyperplane for $C$ passing through $x^*$.

We use notation $CH$ for the convex hull of a set of points, e.g., $C := CH[e(C)]$. The following result holds.

**Lemma 3** Let $C$ be a convex set in $\mathbb{R}^d$ such that $d$ is also its dimension (i.e., Given $a, b \in e(C)$, let $x^* := \frac{1}{2}(a + b)$. Let also $C := CH[e(C) \setminus \{a, b\}]$, while $H_{v,w}$ denotes a supporting hyperplane for $C$ through $x^*$. It holds that, if $S_{a,b} \subset H_{v,w}$, then $x^* \notin C$.

**Proof** By construction $x \in S_{a,b}$. Assume, ad absurdum, $x^* \in C$. Thus, $x^*$ should be a convex combination of the vertices of $C$, i.e.

$$x^* = \sum_{z \in e(C) \setminus \{a, b\}} \lambda_z z, \quad (3)$$

where $\lambda_z \geq 0$ for each $z \in e(C) \setminus \{a, b\}$ and $\sum_{z \in e(C) \setminus \{a, b\}} \lambda_z = 1$. Take the scalar product by $v$:

$$v \cdot x^* = v \cdot \left( \sum_{z \in e(C) \setminus \{a, b\}} \lambda_z z \right) = \sum_{z \in e(C) \setminus \{a, b\}} \lambda_z v \cdot z. \quad (4)$$

By supporting hyperplane definition and simple algebra:

$$v \cdot x^* = \sum_{z \in e(C) \setminus \{a, b\}} \lambda_z v \cdot z \leq \sum_{z \in e(C) \setminus \{a, b\}} \lambda_z v \cdot x^* = v \cdot x^*. \quad (5)$$

This implies $z \in H_{v,w}$ for each $z \in e(C) \setminus \{a, b\}$. As also $a$ and $b$ belong to $H_{v,w}$, we have $e(C) \subset H_{v,w}$. In other words $C$ is included in a hyperplane and it coincides with its boundary, but this is against the original assumption about the dimension of $C$.

As a consequence of this lemma we have that, in Definition 1, the midpoint of the two vertices added to $C$ is a vertex of the new set. This simply following that the two points at minimum (Euclidean) distance belong to a same edge of a convex polytope and the credal set can be always parametrized in order to have full dimension (see discussion in Section 3). When coping with non-Euclidean distances, to have the same result, the two points at minimum distance in Definition 1 should be detected with the additional condition of belonging to a same edge.