Appendix A. Proofs of the main results

**Proposition 21** Consider a set of gambles \( \mathcal{D} \subseteq \mathcal{L} \).

If it is closed under the supremum norm topology, then it satisfies D4. Vice versa, if \( \mathcal{D} \) satisfies also the following property:

\[
f \geq g, \ g \in \mathcal{D} \Rightarrow f \in \mathcal{D}
\]

then D4 implies closure in the supremum norm topology.

**Proof** It is well-known that \( \mathcal{L} \) is a Banach space under the supremum norm and it is a linear topological space (with finite dimension \( n \) in our case) under the topology generated by the supremum norm (see [30]).

Now, consider \( \mathcal{D} \) closed under the supremum norm topology. Then, the limit of every convergent sequence \((f_n)_{n \in \mathbb{N}}\) (with respect to the supremum norm) with \( f_n \in \mathcal{D} \) for every \( n \), must be contained in \( \mathcal{D} \). Consider then, a gamble \( f \) such that \( f + \delta \in \mathcal{D} \) for every \( \delta > 0 \), then \( f + \frac{1}{n} \in \mathcal{D} \) for every \( n \in \mathbb{N}^* \). Its limit w.r.t. the supremum norm is \( f \) and, from the closure of \( \mathcal{D} \), we know that \( f \in \mathcal{D} \).

On the other hand, suppose \( \mathcal{D} \) satisfies D4 and (22). Let us consider a succession \((f_n)_{n \in \mathbb{N}}\) in \( \mathcal{D} \) convergent w.r.t. the supremum norm to a gamble \( f \in \mathcal{L} \). We know that for every \( \varepsilon > 0 \) there exists \( N \in \mathbb{N} \) such that \( \sup |f_n - f| < \varepsilon \) for all \( n \geq N \). In particular, this means that there exist \( h \in \mathcal{L} \) such that:

\[
f_n - f = h^+ - h^-, \ \text{sup } |h| < \varepsilon
\]

hence:

\[
f = (f_n + h^-) - h^+
\]

but, \( f_n + h^- \in \mathcal{D} \) by hypothesis, and \( f = (f_n + h^-) - h^+ \in \mathcal{D} \), from which it follows that \( f + \varepsilon \in \mathcal{D} \). This procedure can be repeated for every \( \varepsilon > 0 \). Then by D4, we have \( f \in \mathcal{D} \).

**Proof** [Proof of Proposition 3] Consider a pair of finite sets \((\mathcal{A}, \mathcal{R})\) for which there exists a coherent set of gambles \( \mathcal{D} \), such that \( \mathcal{D} \supseteq \mathcal{A} \) and \( \mathcal{D} \cap \mathcal{R} = \emptyset \). Then, the minimal coherent set \( \mathcal{D} \) that satisfies these conditions is \( \overline{\mathcal{D}(\mathcal{A})} : = \text{posi}(\mathcal{A} \cup \mathcal{R}) \), where \( \text{posi}(\mathcal{K}) := \left\{ \sum_{j=1}^{k} \lambda_j f_j : f_j \in \mathcal{K}, \lambda_j > 0, r \geq 1 \right\} \) for every \( \mathcal{K} \subseteq \mathcal{L}(\Omega) \) and where \( \mathcal{R}^\mathcal{A} \) of a set \( \mathcal{X} \subseteq \mathcal{L} \) represents the closure of \( \mathcal{X} \) with respect to the supremum norm topology. In fact, \( \mathcal{D}(\mathcal{A}) \) is clearly the minimal set \( \mathcal{D} \) that satisfies D1 - D3 such that \( \mathcal{D} \supseteq \mathcal{A} \). Then, thanks to Proposition 21, \( \overline{\mathcal{D}(\mathcal{A})} \) is the minimal coherent set \( \mathcal{D} \) such that \( \mathcal{D} \supseteq \mathcal{A} \) and clearly, by hypothesis, we know also that \( \overline{\mathcal{D}(\mathcal{A})} \cap \mathcal{A} = \emptyset \). This fact is also well-known in literature [30].

However, \( \overline{\mathcal{D}(\mathcal{A})} \), by definition, is a polyhedral (convex) cone [1, Definition 2.3.2]. Indeed \( \overline{\mathcal{D}(\mathcal{A})} \) can be rewritten as:

\[
\overline{\mathcal{D}(\mathcal{A})} = \overline{\text{posi}(\mathcal{A} \cup \mathcal{R})} = \sum_{j=1}^{N} (\phi_j(g))^T \beta_j = \sum_{j=1}^{N} (\mathbb{1}_{|\mathcal{A}_j|} g)^T \beta_j = K \beta_k = Km,
\]

where, for every \( j \), \( \mathcal{A}_j \) are the partitions of the type 4 with \( \omega_j = \beta_j \) and \( g^T \beta_k = m \), for all \( k = 1, \ldots, N \), with \( 1 \leq K \leq N \).

\[
C \seteq \left\{ g : g = \sum_{j=1}^{r} \lambda_j f_j, \ f_j \in (\mathcal{A} \cup \{I_{\mathbb{N}}\}_r), r \geq 1, \lambda_j \geq 0 \right\}
\]

where the last equality derives from the facts that \( \mathcal{D}(\mathcal{A}) = \text{posi}(\mathcal{A} \cup \mathcal{R}) \) is generated by the finite set \((\mathcal{A} \cup \{I_{\mathbb{N}}\}_r)\); \( \mathcal{C} \) is already closed under the usual topology of \( \mathbb{R}^n \) that coincides with the closure with respect to the supremum norm topology, for every topological space with \( n \) dimension [30, Appendix D]. The latter is true because, thanks to the Minkowsky-Weyl theorem [1], we know that \( C \) is an intersection of a finite number of closed halfspaces whose bounding hyperspaces pass through the origin:

\[
C = \{ g : g^T \beta_j \geq 0, \ j = 1, \ldots, N \}
\]
Hence, \( g \) is classified in the same way by the classifiers \( PLC() \) and \( LCG() \). Therefore, in particular, if \( g \in (\mathcal{A} \cup \mathcal{U}) \), \( m \geq 0 \) and hence \( \sum_{j=1}^{N} (\phi_j(g))^T \beta_j = Km \geq 0 \), if \( \text{instead} \ g \in (\mathcal{A} \cup \mathcal{U}) \) then \( m < 0 \) and hence \( \sum_{j=1}^{N} (\phi_j(g))^T \beta_j = Km < 0 \).

Vice versa, let us consider a \( \Phi \)-separable pair \( (\mathcal{A} \cup T, \mathcal{U} \cup F) \) and let us suppose the existence of a classifier \( LCG() \in LCG(\mathcal{A} \cup T, \mathcal{U} \cup F) \) with parameters \( a_j = \beta_j^T \), for all \( j = 1, \ldots, N \). Let us define \( m' = \min(g^T \beta_1', \ldots, g^T \beta_N') \).

Then, for any \( g \in \mathcal{L} \) we have:

\[
\sum_{j=1}^{N} (\phi_j(g))^T \beta_j' = \sum_{k=1}^{K} g^T \beta_k' = Km',
\]

where again \( g^T \beta_k' = m', \) for all \( k = 1, \ldots, K \), with \( 1 \leq K \leq N \). Let us consider a binary piecewise linear classifier \( PLC() \) with parameters \( \{\beta_j'\}_{j=1}^{N} \). Then, again, \( g \) is classified in the same way by the classifiers \( LCG() \) and \( PLC() \). This is in particular true for \( g \in (\mathcal{A} \cup T) \) and \( g \in \mathcal{U} \cup F \). This means also that \( \beta_j' \geq 0 \), for all \( j = 1, \ldots, N \).

**Lemma 22** If a set \( \mathcal{D} \subseteq \mathcal{L} \) satisfies \( D1, D3^* \) and \( D4 \) then it satisfies (22).

**Proof** Consider \( f \geq g \) with \( g \in \mathcal{D} \). Then \( f = g + t \) with \( t \in T \). For any \( \varepsilon > 0 \), \( f + \varepsilon = g + t + \varepsilon \). Moreover, we can always find \( \lambda \in (0, 1) \) such that \( \lambda g \leq f + \varepsilon \).

Therefore, we have \( f + \varepsilon = \lambda g + (1 - \lambda)(\frac{g + \varepsilon - \lambda g}{1 - \lambda}) \).

Now, \( g \in \mathcal{D} \) by hypothesis and \( \frac{g + \varepsilon - \lambda g}{1 - \lambda} \in T \), so \( f + \varepsilon \in \mathcal{D} \). This can be repeated for every \( \varepsilon > 0 \), then \( f + \varepsilon \in \mathcal{D} \) for all \( \varepsilon > 0 \) that implies, by D4, that \( f \in \mathcal{D} \).

**Lemma 23** Given a pair of finite sets \( (\mathcal{A}, \mathcal{R}) \) for which there exists a convex coherent set of gambles \( \mathcal{D} \) such that \( \mathcal{D} \supseteq \mathcal{A} \) and \( \mathcal{D} \cap \mathcal{R} = \emptyset \), then the minimal such set \( \mathcal{D} = ch(\mathcal{A} \cup T) \).

**Proof** \( ch(\mathcal{A} \cup T) \) satisfies D1 by definition and D3* [24, Theorem 6.2] and D4, thanks to Proposition 21.

Let us indicate with \( D(\mathcal{A}, \mathcal{R}) \), the class of convex coherent sets of gambles \( \mathcal{D} \) such that \( \mathcal{D} \supseteq \mathcal{A} \) and \( \mathcal{D} \cap \mathcal{R} = \emptyset \). Thanks to Lemma 22 and Proposition 22, every \( \mathcal{D} \in D(\mathcal{A}, \mathcal{R}) \), is a convex closed set (with respect to the topology of \( \mathbb{R}^n \) or equivalently respect to the supremum norm topology) that contains \( (\mathcal{A} \cup T) \).

Given the fact that \( ch(\mathcal{A} \cup T) \supseteq (\mathcal{A} \cup T) \) and, by definition, it is the intersection of all the closed (with respect to the topology of \( \mathbb{R}^n \) or equivalently respect to the supremum norm topology) and convex sets containing \( (\mathcal{A} \cup T) \), we have that \( ch(\mathcal{A} \cup T) \supseteq \mathcal{D} \), for all \( \mathcal{D} \in D(\mathcal{A}, \mathcal{R}) \).

But, every \( \mathcal{D} \in D(\mathcal{A}, \mathcal{R}) \), satisfies \( \mathcal{D} \cap (\mathcal{U} \cup F) = \emptyset \). Therefore, \( ch(\mathcal{A} \cup T) \cap (\mathcal{U} \cup F) = \emptyset \), and hence it is also the smallest set \( \mathcal{D} \in D(\mathcal{A}, \mathcal{R}) \). This concludes the proof.

**Lemma 24** Consider a finite set \( \mathcal{A} \subseteq \mathcal{L} \). Then:

\[
ch(\mathcal{A} \cup T) = ch^+(\mathcal{A} \cup \{0\}) := \{ g : g \geq f, f \in ch(\mathcal{A} \cup \{0\}) \}.
\]

**Proof** First of all, we can observe that:

\[
ch^+(\mathcal{A} \cup \{0\}) = \{ g : g \geq f, f \in ch(\mathcal{A} \cup \{0\}) \} = \sum_{i=1}^{N} \alpha_i g_i + \sum_{j=1}^{N} \gamma_j e_j =: ch(\mathcal{A} \cup \{0\}) + pos(\epsilon_1, \ldots, \epsilon_n)
\]

with \( I, J \) finite, \( g_i \in \mathcal{A} \cup \{0\} \), \( \alpha_i, \gamma_j \geq 0 \) and \( \sum \alpha_i = 1 \), where \( \epsilon_i \) is the canonical basis in \( \mathbb{R}^n \) and \( pos(\epsilon_1, \ldots, \epsilon_n) \) is a convex polyhedral cone. From [27, Corollary 7.1.b], it follows that \( ch^+(\mathcal{A} \cup \{0\}) \) is a convex (closed) polyhedron. Hence \( ch^+(\mathcal{A} \cup \{0\}) = ch^+(\mathcal{A} \cup \{0\}) \). Now, we divide the proof in two parts:

- \( ch(\mathcal{A} \cup T) \subseteq ch^+(\mathcal{A} \cup \{0\}) \). Notice that, thanks to the previous observation, it is sufficient to show that \( ch(\mathcal{A} \cup T) \subseteq ch^+(\mathcal{A} \cup \{0\}) \). So, let us consider \( g \in ch(\mathcal{A} \cup T) \). By definition, we have:

\[
g = \sum_{k=1}^{r} \lambda_k g_k
\]

with \( \lambda_k \geq 0 \), for all \( k = 1, \ldots, r \), \( r \geq 1 \), \( \sum \lambda_k = 1 \), \( g_k \in ch(\mathcal{A} \cup T) \). Let us indicate with \( Ind_{A \cup T} := \{ k \in \{1, \ldots, r\} \text{ such that: } g_k \in \mathcal{A} \cup T \} \) and \( Ind_T := \{ k \in \{1, \ldots, r\} \text{ such that: } g_k \in T \} \). Then we have:

\[
g \geq \sum_{k \in Ind_{A \cup T}} \lambda_k g_k + \sum_{k \in Ind_T} \lambda_k 0,
\]

hence \( g \in ch^+(\mathcal{A} \cup \{0\}) \).

- \( ch^+(\mathcal{A} \cup \{0\}) \subseteq ch(\mathcal{A} \cup T) \). By definition, \( ch(\mathcal{A} \cup T) \) is a closed convex set that contains \( T \). Therefore, from Proposition 21 and Lemma 22, we have:

\[
ch(\mathcal{A} \cup \{0\}) \subseteq ch(\mathcal{A} \cup T) \Rightarrow ch^+(\mathcal{A} \cup \{0\}) \subseteq ch(\mathcal{A} \cup T).
\]

**Proof** [Proof of Proposition 8] Consider a pair of sets \( (\mathcal{A}, \mathcal{R}) \) for which there exists a convex coherent set of gambles \( \mathcal{D} \), such that \( \mathcal{D} \supseteq \mathcal{A} \) and \( \mathcal{D} \cap \mathcal{R} = \emptyset \). Then the minimal convex coherent set \( \mathcal{D} \), which satisfies these conditions, is \( ch(\mathcal{A} \cup T) \). Thanks to Lemma 24, we know that it can be rewritten as:

\[
ch(\mathcal{A} \cup T) = ch^+(\mathcal{A} \cup \{0\}),
\]

where \( ch^+(\mathcal{A} \cup \{0\}) \) is a convex polyhedron. Any convex polyhedron can be written as an intersection of hyperspaces, whose border is a piecewise affine function. Therefore, there exists a piecewise affine classifier \( PAC() \), such
that $\text{ch}(\mathcal{A} \cup T) = \text{ch}^+(\mathcal{A} \cup \{0\}) = \{g : \text{PAC}(g) = 1\}$. Note moreover that $\text{ch}(\mathcal{A} \cup T) = \{g : \text{PAC}(g) = 1\} \supseteq (\mathcal{A} \cup T)$ and $(\mathcal{A} \cup T) = \{g : \text{PAC}(g) = 1\} \cap (\mathcal{A} \cup T) = \emptyset$ by construction.

Vice versa, consider a piecewise affine separable pair $(\mathcal{A} \cup T, \mathcal{R} \cup F)$. Let us consider a piecewise affine classifier $\text{PAC}(-) \in \text{PAC}(\mathcal{A} \cup T, \mathcal{R} \cup F)$. Now, the set:

$$\mathcal{D} := \{g : \text{PAC}(g) = 1\} = \{g : g^T \beta_j + \alpha_j \geq 0, \text{ for all } j = 1, \ldots, N\}$$

for some $\beta_j \in \mathbb{R}^n$ with $\beta_j \geq 0$ and $\alpha_j \in \mathbb{R}$ for all $j \in \{1, \ldots, N\}$, is a convex coherent set of gambles such that $\mathcal{D} \supseteq \mathcal{A}$ and $\mathcal{D} \cap \mathcal{R} = \emptyset$. Indeed:

- $\mathcal{T} \subseteq \mathcal{D}$ and $\mathcal{D} \cap \mathcal{R} = \emptyset$, by definition, hence it satisfies D1 and D2; 
- $\mathcal{D}$ satisfies D3. Consider $g_1, g_2 \in \mathcal{D}$. Then $t g_1 + (1-t)g_2 \in \mathcal{D}$, for all $t \in [0, 1]$. Indeed, 

$$(tg_1 + (1-t)g_2)^T \beta_j + \alpha_j = (tg_1)^T \beta_j + ((1-t)g_2)^T \beta_j + t \alpha_j + (1-t)\alpha_j = t((g_1)^T \beta_j + \alpha_j) + (1-t)(g_2)^T \beta_j + \alpha_j \geq 0$$

for all $j \in \{1, \ldots, N\}$.

- $\mathcal{D}$ is closed in the usual topology of $\mathbb{R}^n$ because it is the intersection of a finite number of closed half-spaces hence, thanks to Proposition 21, it satisfies D4. Clearly, by the fact that $\text{PAC}(-) \in \text{PAC}(\mathcal{A} \cup T, \mathcal{R} \cup F)$, it is also true that $\mathcal{A} \subseteq \mathcal{D}$ and $\mathcal{D} \cap \mathcal{R} = \emptyset$. 

\section*{Proof [Proof of Proposition 10]}

Consider a piecewise affine separable pair $(\mathcal{A} \cup T, \mathcal{R} \cup F)$ and a classifier $\text{PAC}(-) \in \text{PAC}(\mathcal{A} \cup T, \mathcal{R} \cup F)$ with parameters $\{\beta_j, \alpha_j\}_{j=1}^N$.

Then, a classifier $\text{LC}_\mathcal{Y}(-)$ of the type (11) with parameters $\alpha_j = \beta_j / \alpha_j$, for all $j = 1, \ldots, N$, classifies $\mathcal{A} \cup T$ as 1 and $\mathcal{R} \cup F$ as -1. Indeed, consider $g \in \mathcal{L}$ and let us define $m := \min(g^T \beta_1 + \alpha_1, \ldots, g^T \beta_N + \alpha_N)$. Then,

$$\sum_{j=1}^N (\psi_j(g))^T \beta_j / \alpha_j = \sum_{j=1}^N \left(1_{\mathcal{R}_j} \left(\left[\begin{array}{c} g \\ 1 \end{array}\right]\right) \right)^T \beta_j / \alpha_j = \sum_{k=1}^K (g^T \beta_k + \alpha_k) = Km,$$

where, for every $j$, $\mathcal{R}_j$ are the partitions of the type 10 with $\alpha_j = \beta_j$ and $g^T \beta_k + \alpha_k = m$, for any $k = 1, \ldots, K$, with $1 \leq K \leq N$. Hence, $g$ is classified in the same way by the classifiers $\text{PAC}(-)$ and $\text{LC}_\mathcal{Y}(-)$. Therefore, in particular, if $g \in (\mathcal{A} \cup T)$, $m \geq 0$ and hence $\sum_{j=1}^N (\psi_j(g))^T \beta_j / \alpha_j = Km \geq 0$, if instead $g \in (\mathcal{R} \cup F)$ then $m < 0$ and hence $\sum_{j=1}^N (\psi_j(g))^T \beta_j / \alpha_j < 0$.

\section*{Lemma 25}

Given a pair of finite sets $(\mathcal{A}, \mathcal{R})$ for which there exists a positive additive coherent set of gambles $\mathcal{D}$, such that $\mathcal{D} \supseteq \mathcal{A}$ and $\mathcal{D} \cap \mathcal{R} = \emptyset$, then the smallest such set is:

$$\mathcal{D} = \{g : (\exists f \in \mathcal{A} \cup \{0\}) g \geq f\}.$$ 

\section*{Proof [Proof of Proposition 13]}

Consider a pair of sets $(\mathcal{A}, \mathcal{R})$ for which there exists a positive additive coherent set of gambles $\mathcal{D}$, such that $\mathcal{D} \supseteq \mathcal{A}$ and $\mathcal{D} \cap \mathcal{R} = \emptyset$. Then the minimal such set is $\uparrow (\mathcal{A} \cup \{0\})$. However, it can be rewritten as:

$$\uparrow (\mathcal{A} \cup \{0\}) = \{g \in \mathcal{L} : \text{PWPC}(g) = 1\}$$

where PWPC(-) is a PWP classifier, defined as:

$$\text{PWPC}(g) := \begin{cases} 
1 & \text{if } \exists f^j \in (\mathcal{A} \cup \{0\}) \text{ s.t. } g \geq f^j, \\
-1 & \text{otherwise}. 
\end{cases}$$

Therefore, given that $\mathcal{A} \cup T \subseteq \uparrow (\mathcal{A} \cup \{0\})$ and $(\uparrow (\mathcal{A} \cup \{0\}) = \{g : \text{PWPC}(g) = 1\}$ and $(\uparrow (\mathcal{A} \cup \{0\}) = \{g : \text{PWPC}(g) = 1\}$.
1}) \cap (\mathcal{R} \cup F) = \emptyset$, we have that $(\mathcal{A} \cup T, \mathcal{R} \cup F)$ is PWPC separable. Vice versa, consider a PWPC separable pair $(\mathcal{A} \cup T, \mathcal{R} \cup F)$ and a classifier PWPC$(\cdot) \in \text{PWPC}(\mathcal{A} \cup T, \mathcal{R} \cup F)$. Then:

$$\mathcal{D} := \{ g : \text{PWPC}(g) = 1 \}$$

is, by construction, a positive additive coherent set of gambles. Indeed, it clearly satisfies D1, D2, D3**. Further, it is closed because it is a finite intersection of closed sets (respect to the usual topology of $\mathbb{R}^n$) hence, by Proposition 21, it satisfies D4. It satisfies also $\mathcal{D} \supseteq \mathcal{A}$ and $\mathcal{D} \cap \mathcal{R} = \emptyset$ by hypothesis.

**Proof** [Proof of Proposition 15] Consider a PWPC separable pair $(\mathcal{A} \cup T, \mathcal{R} \cup F)$ and a classifier PWPC$(\cdot) \in \text{PWPC}(\mathcal{A} \cup T, \mathcal{R} \cup F)$ with parameters $\mathcal{F} = \{ f_j \}_{j=1}^N$.

Then, a classifier $LC_p(\cdot)$ of the type (14), with parameters $\omega^j = f^j$ and $\beta_{ij}^j = \begin{bmatrix} 1 \\ \vdots \\ -f_i^j \\ \vdots \\ -f_n^j \end{bmatrix}$ for all $j = 1, \ldots, N$, classifies $\mathcal{A} \cup T$ as 1 and $\mathcal{R} \cup F$ as -1.

Indeed, consider $g \in \mathcal{L}'$ and let us define $m := \max_k (\min_i (g_i - f_i^k))$. Then:

$$\sum_{j=1}^N (\rho_j(g))^T \beta_j' = \sum_{j=1}^N \sum_{i=1}^n \gamma_j(g_i - f_i^j) = Klm$$

where, for every $i, j$, $\zeta_{ij}$ are the partitions of the type 13 with $\omega^j = f^j$ and where $1 \leq L \leq n$, $1 \leq K \leq N$. Hence, $g$ is classified in the same way by the classifiers PWPC$(\cdot)$ and $LC_p(\cdot)$. Therefore, in particular, if $g \in (\mathcal{A} \cup T)$, $m \geq 0$ and hence $LC_p(g) = 1$. If instead $g \in (\mathcal{R} \cup F)$ then $m < 0$ and hence $LC_p(g) = -1$.

Vice versa, let us consider a P-separable pair $(\mathcal{A} \cup T, \mathcal{R} \cup F)$ and let us suppose the existence of a classifier $LC_p(\cdot) \in \text{LC}_p(\mathcal{A} \cup T, \mathcal{R} \cup F)$ with parameters $\{ \beta_{ij}^j \}_{j=1}^N$ such that $\beta_{ij}^j > 0$, and $\omega^j = -\frac{\beta_{ij}^j}{\beta_{ij}'}$ for all $i = 1, \ldots, n, j = 1, \ldots, N$. Let us define $m' := \max_k (\min_i (g_i - (-\frac{\beta_{ij}^j}{\beta_{ij}'})$. Then, for any $g \in \mathcal{L}'$:

$$\sum_{j=1}^N (\rho_j(g))^T \beta_j' = \sum_{j=1}^N \sum_{i=1}^n \gamma_j(g_i - f_i^j) + \beta_{ij}' g_i + \beta_{ij}^j n) =$$

$$\sum_{j=1}^N \sum_{i=1}^n \beta_{ij}' \gamma_j(g_i - (-\frac{\beta_{ij}^j}{\beta_{ij}'})) = m' K \sum_{j=1}^L \sum_{i=1}^K \beta_{ij}'$$

with $1 \leq K \leq N$, $1 \leq L \leq n$. Let us consider a PWPC classifier PWPC$(\cdot)$ with parameters $\mathcal{F} = \{ f_j \}_{j=1}^N$, such that $f_i^j = -\frac{\beta_{ij}^j}{\beta_{ij}'}$, for all $i, j$. Then, again, $g$ is classified in the same way by the classifiers $LC_p(\cdot)$ and PWPC$(\cdot)$. This is in particular true for $g \in \mathcal{A} \cup T$ and $g \in \mathcal{R} \cup F$. 

---

**Nonlinear Desirability**