Same-Decision Probability:
A New Tool for Decision Making

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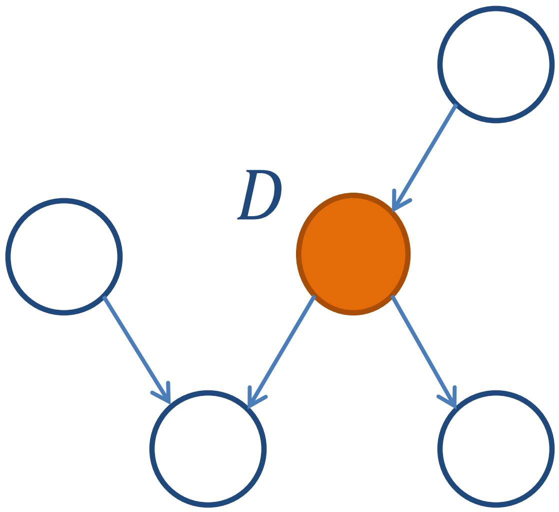
Introduction

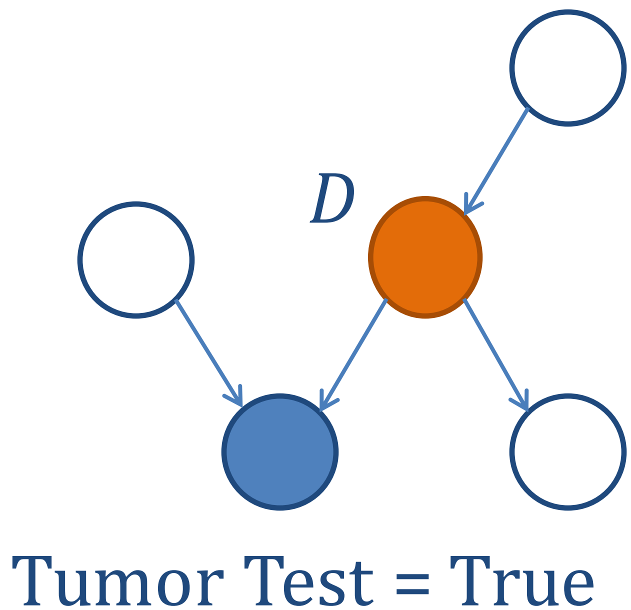
Bayesian Network *N*



- We make a decision based on *D*.
- Example: *D* – Health state of a patient.
- Patient healthy: *D* = True
- Patient unhealthy: *D* = False

Introduction (2)

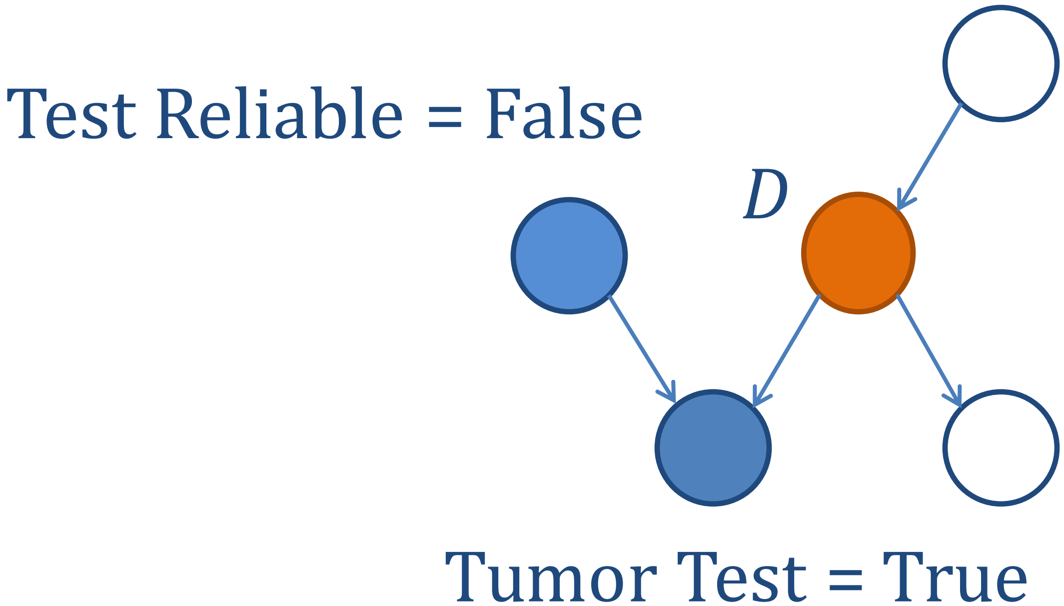
Bayesian Network *N*



Diagnosis: Patient is sick.

Introduction (3)

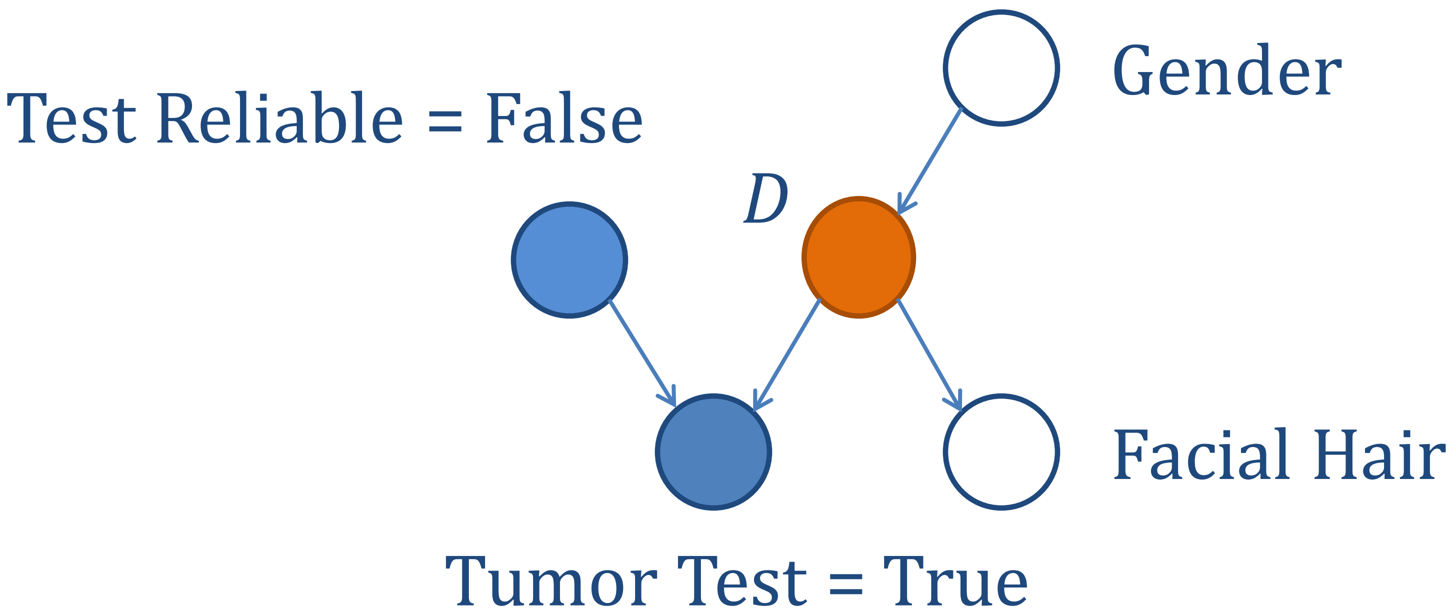
Bayesian Network *N*



Diagnosis: Patient is healthy

Stopping Criteria

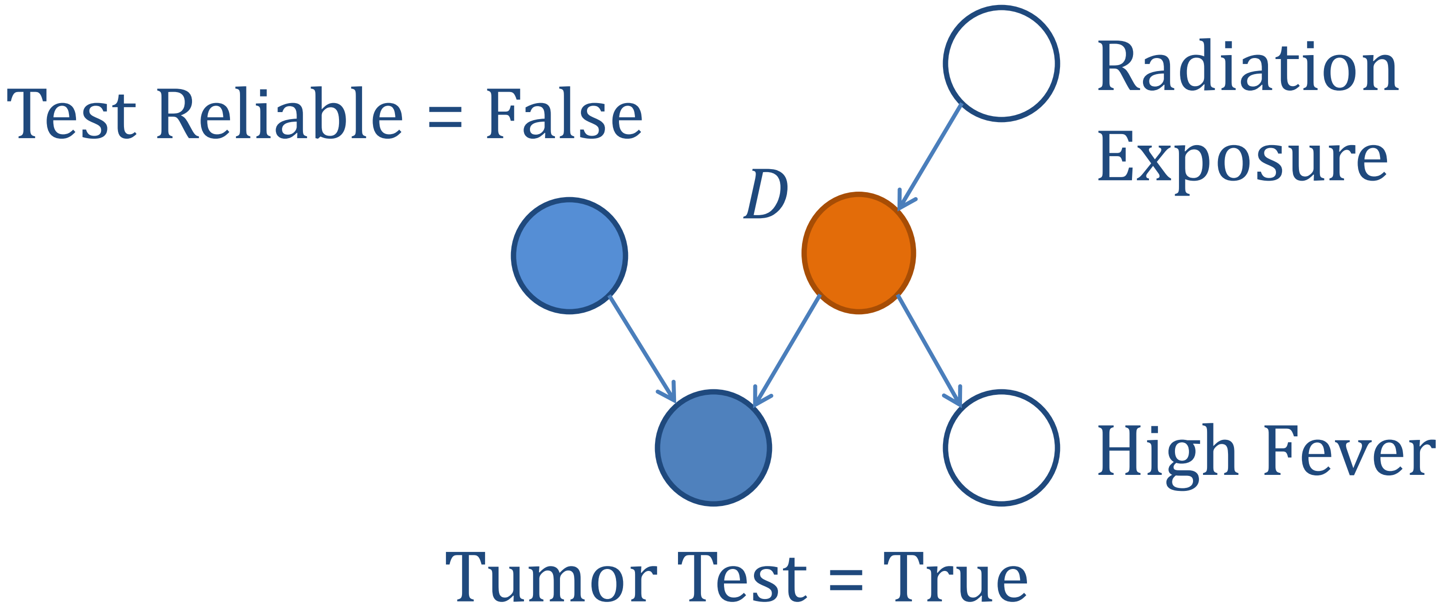
Bayesian Network *N*



Diagnosis: Patient is healthy

Stopping Criteria (2)

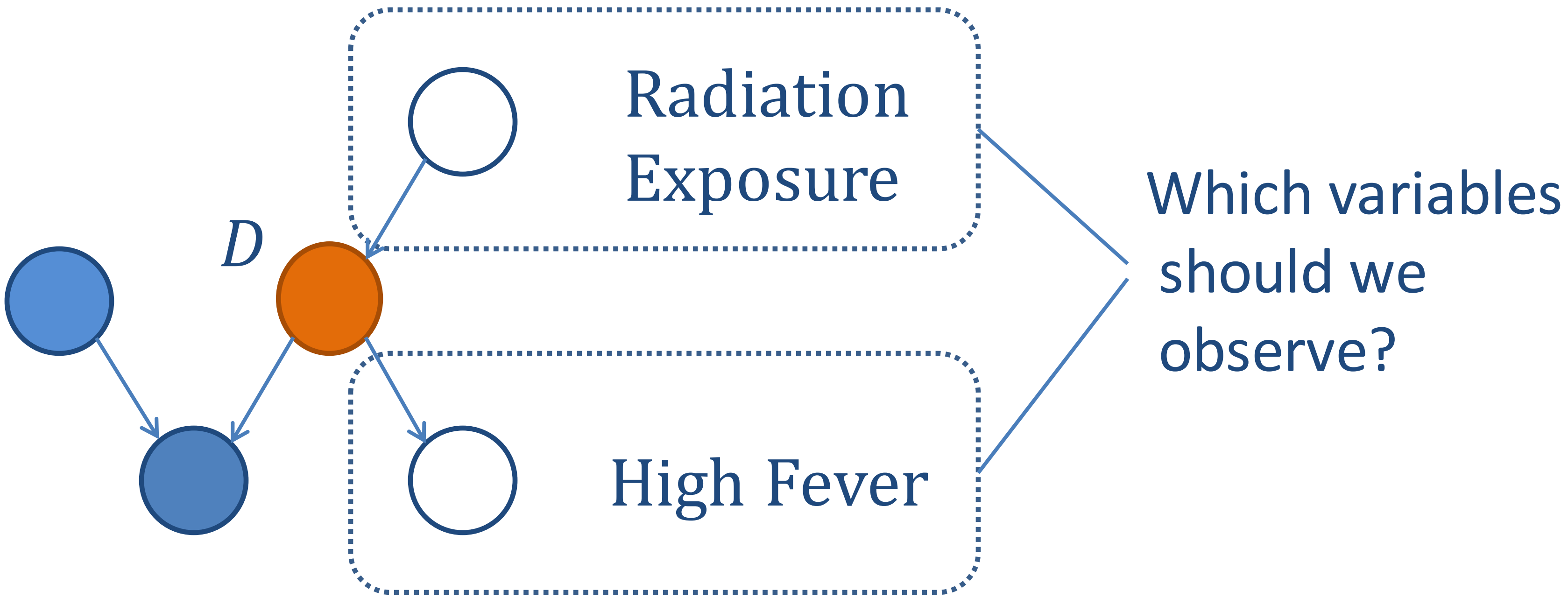
Bayesian Network *N*



Diagnosis: Patient is healthy

Selection Criteria

Bayesian Network N



Diagnosis: Patient is healthy

Decision Tools

Current Decision Tools

Stopping Criteria

- Expend budget for observation.
- $\Pr(D=d|\mathbf{e}) \geq T$
- Value of information of observations > cost.

Selection Criteria

- Entropy reduction
- Margins of confidence
- Utility (influence diagram setting).

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New Decision Tools

- *Same-decision Probability*
- *Same-decision Probability*

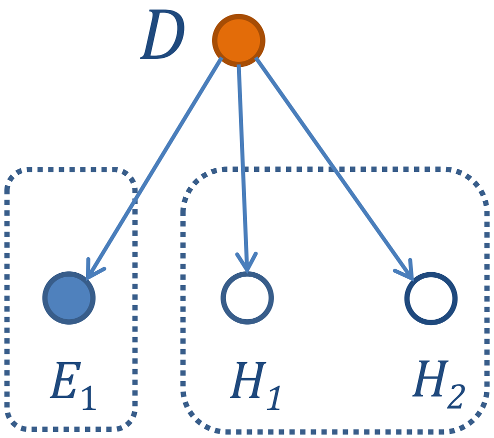
Same-Decision Probability

Same-Decision Probability - probability that we would have made the same decision had we known some additional variables.

- Useful as a stopping criteria.
- Useful as a selection criteria.

Same-Decision Probability Example

	θ_D
D=T	0.60
D=F	0.40



	D=T	D=F
*=T	0.70	0.30
*=T	0.30	0.70

- Naive Bayes Classifier with missing features
- E_1 = True
- Two features, H_1 and H_2 unobserved.
- Pr(D=T|**e**) = 0.778

Same-Decision Probability Example

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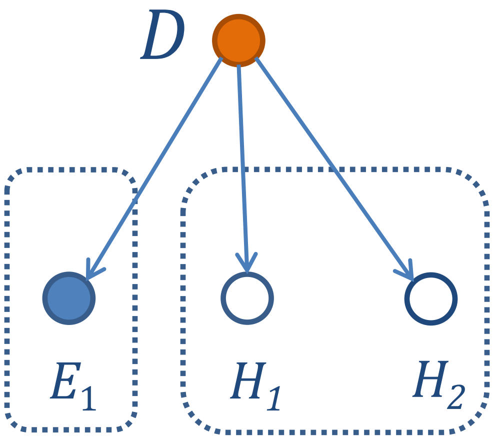

	D=T	D=F
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- Pr(D=T|**e**) = 0.778

H_1	H_2	Pr(h|e)	Pr(D=T|h,e)
T	**T**	**0.401**	**0.95**
T	**F**	**0.21**	**0.778**
F	**T**	**0.21**	**0.778**
F	F	0.179	0.39

SDP is calculated to be 0.401 + 0.21 + 0.21 = 0.821.

Same-Decision Probability Definition

The *SDP* over variables **H,** with a decision function *F*, interest variable *D*, and evidence **e**, is defined as:

$$\mathrm{SDP}(F, D, \mathbf{H}, \mathbf{e}) = \Sigma_{\mathbf{h}} \, [F(\Pr(D \mid \mathbf{h}, \mathbf{e}))]_{\mathbf{h}} \Pr(\mathbf{h} \mid \mathbf{e})$$

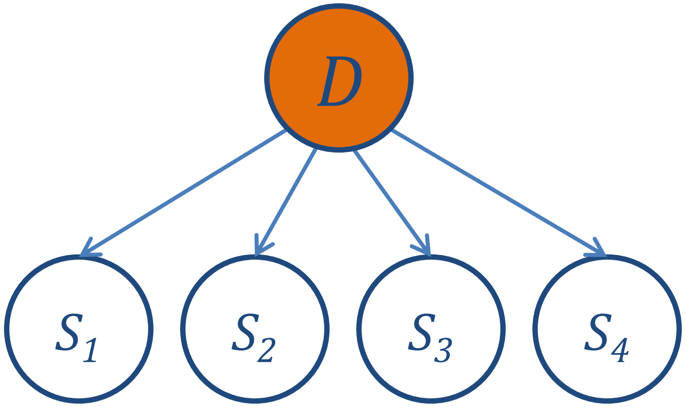
$[.]_{\mathbf{h}}$ – indicator function

- 1 when F(Pr (*D* | **h**, **e**)) = F(Pr (*D* | **e**))
- 0 otherwise

SDP – Stopping Criteria

- Calculating SDP can act as a stopping criteria.
  - Provides a quantitative measure of how likely our decision is to change if some unobserved variables were known.
  - Can tell us when no other further observations are necessary.

SDP – Stopping Criteria Example



Threshold-based decision:

$$\Pr(D{=}+ \mid \mathbf{e}) \geq 0.55$$

	$D = +$	$D = -$
$S_1 = +$	0.55	0.45
$S_1 = -$	0.45	0.55

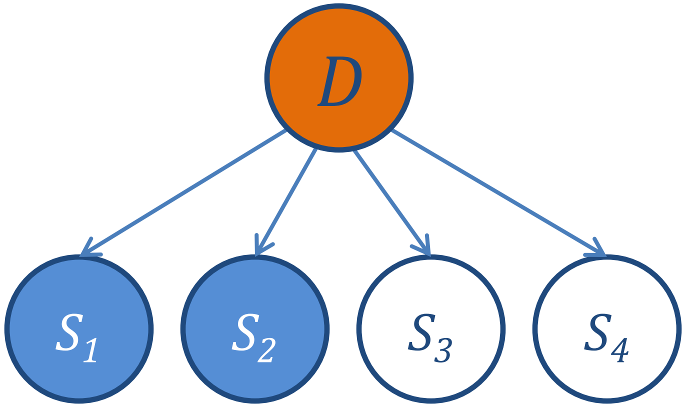
	$D = +$	$D = -$
$S_2 = +$	0.55	0.45
$S_2 = -$	0.45	0.55

	θ_D
$D{=}+$	0.50
$D{=}-$	0.50

	$D = +$	$D = -$
$S_3 = +$	0.60	0.40
$S_3 = -$	0.40	0.60

	$D = +$	$D = -$
$S_4 = +$	0.65	0.35
$S_4 = -$	0.35	0.65

SDP – Stopping Criteria Example



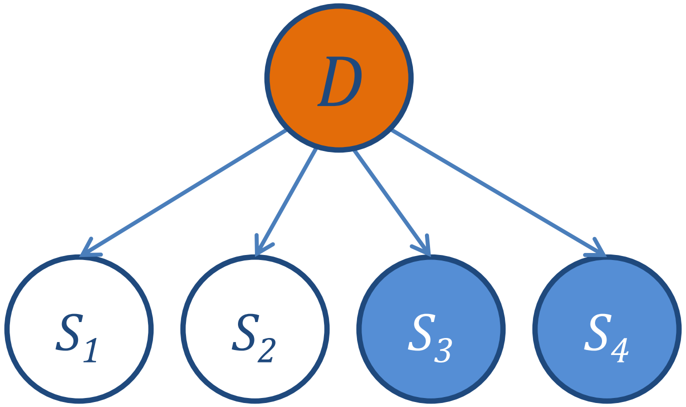
CASE 1

S_1 and S_2 are observed to be +.

- Pr(D=+| S_1=+, S_2=+) = 0.60

SDP over S_3 and S_4: **0.53**

SDP – Stopping Criteria Example



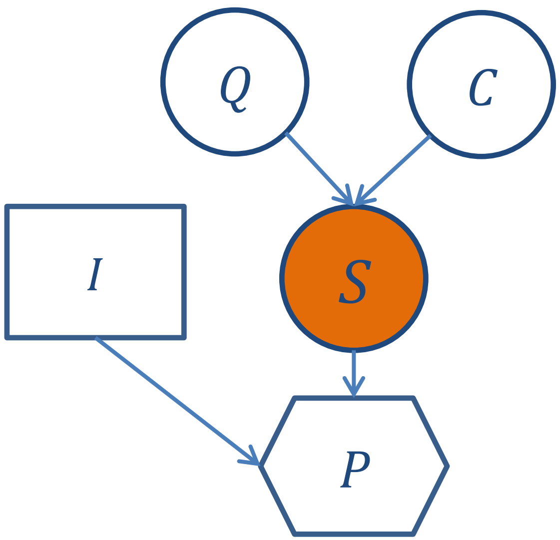
CASE 2

S_3 and S_4 are observed to be +.

- Pr(D=+| S_1=+, S_2=+) = 0.74

SDP over S_1 and S_2: **1.0**

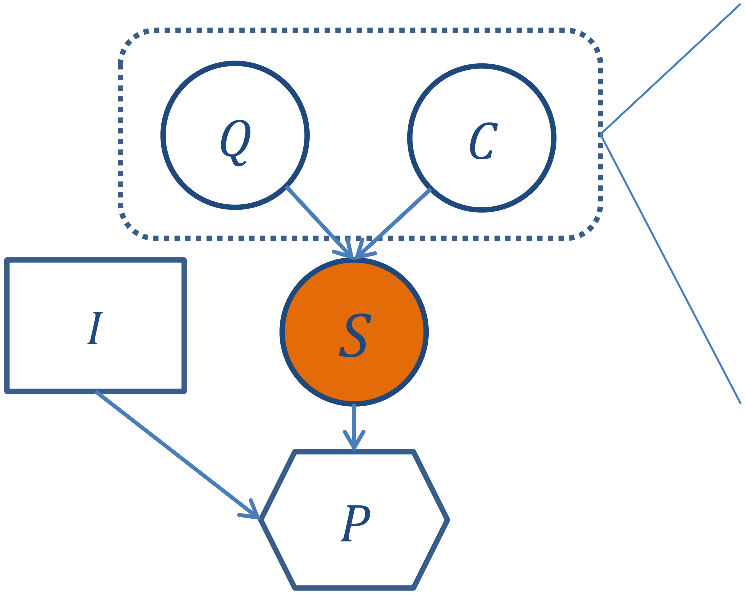
SDP – Stopping Criteria Example (2)



Influence diagram modeling a startup company investment problem:

- ***I***={T,F} is the decision node; represents our choice on whether or not to invest.
- ***P*** (Profit) is the value node.
- ***S***={T,F} is whether or not the startup will succeed.
- ***Q***={T,F} is whether or not the startup having a quality idea.
- ***C***={T,F} is whether or not the existing competition is successful.

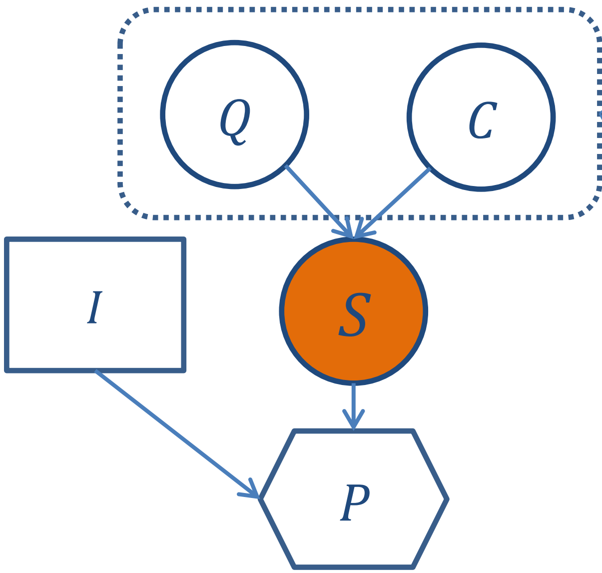
SDP – Stopping Criteria Example (2)



Case 1: Value of observing *Q* and *C* is $680,000.

Case 2: Value of observing *Q* and *C* is $680,000.

SDP – Stopping Criteria Example (2)



Case 1: Value of observing *Q* and *C* is $680,000.

Low Risk, Low Reward

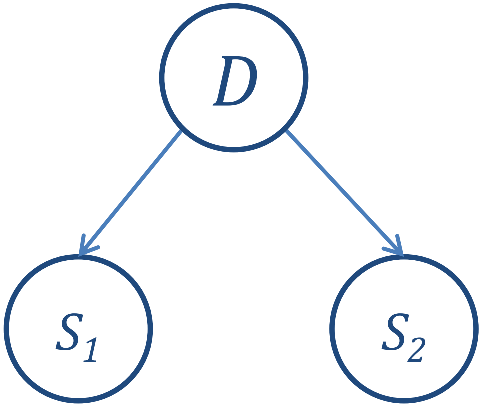
SDP – 0.60

Case 2: Value of observing *Q* and *C* is $680,000.

High Risk, High Reward

SDP – 0.99

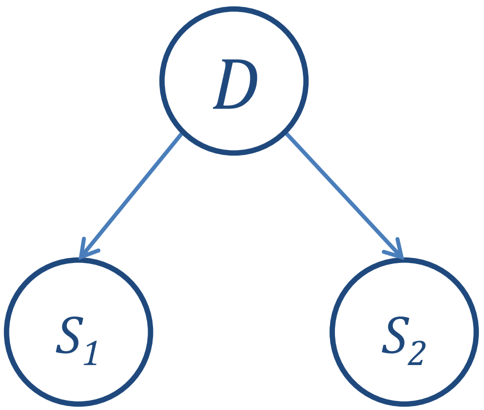
SDP – Selection Criteria Example



Threshold-based decision: $\Pr(D{=}+ \mid \mathbf{e}) \geq 0.80$

Problem: If S_1 and S_2 are unobserved, and only one observation is allowed, which should be observed next?

SDP – Selection Criteria Example



	θ_D
D=+	0.50
D=–	0.50

	D = +	D = -
S_1 = +	0.80	0.20
S_1 = -	0.20	0.80

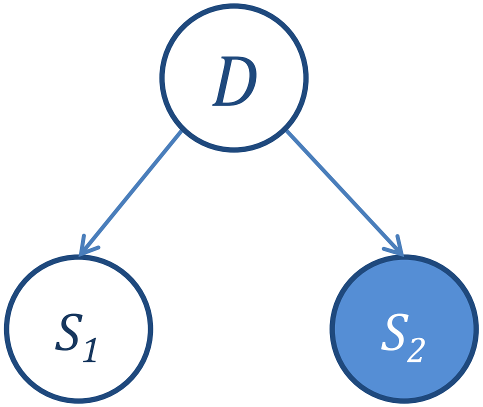
	D = +	D = -
S_2 = +	0.75	0.05
S_2 = **o**	0.20	0.20
S_2 = -	0.05	0.75

Pr(D=+) < 0.80

Threshold not crossed.

SDP – Selection Criteria Example

SDP of observing S_2

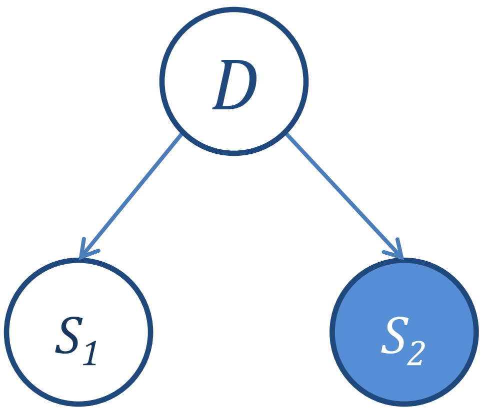


Case 1: S_2 observed to be +

SDP is 0.7625

SDP – Selection Criteria Example

SDP of observing S_2



Case 1: S_2 observed to be +

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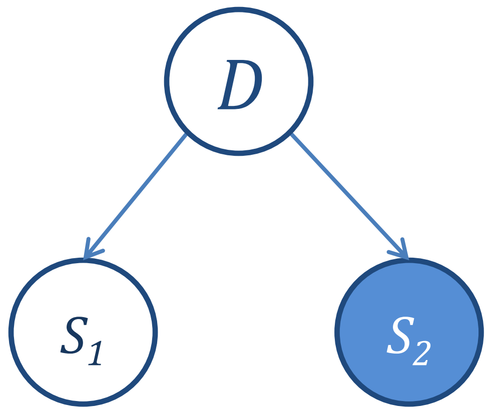
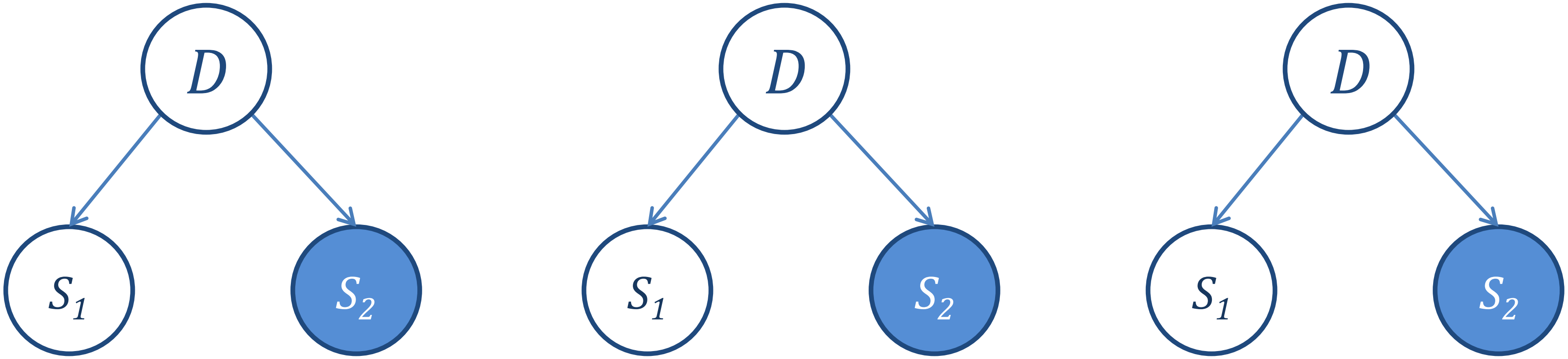


Case 2: S_2 observed to be **o**

SDP is 0.5

SDP – Selection Criteria Example

SDP of observing S_2



Case 1: S_2 observed to be +

SDP is 0.7625

Case 2: S_2 observed to be **o**

SDP is 0.5

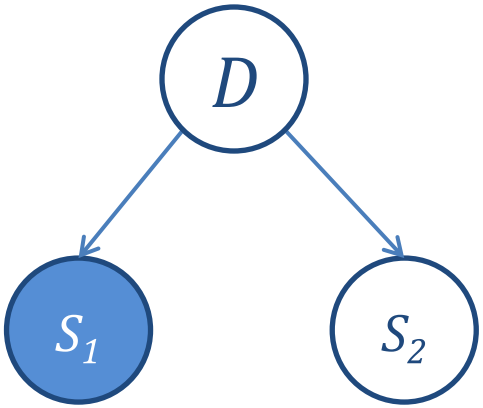
Case 3: S_2 observed to be –

SDP is 1.0

Expected SDP of observing S_2: **0.805**

SDP – Selection Criteria Example

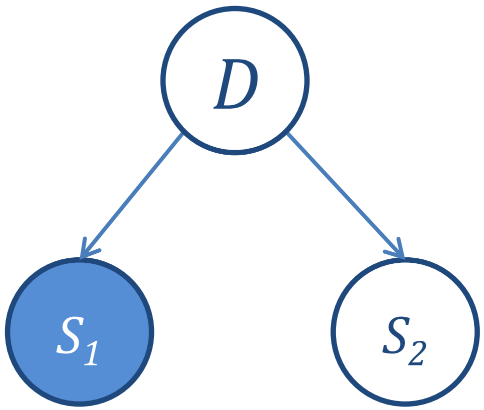
SDP of observing S_1



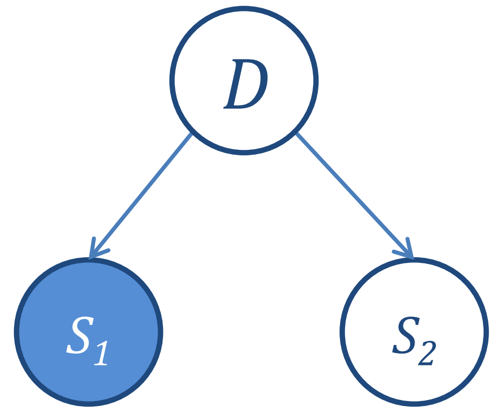
Case 1: S_1 observed to be –
SDP is 1.0

SDP – Selection Criteria Example

SDP of observing S_1



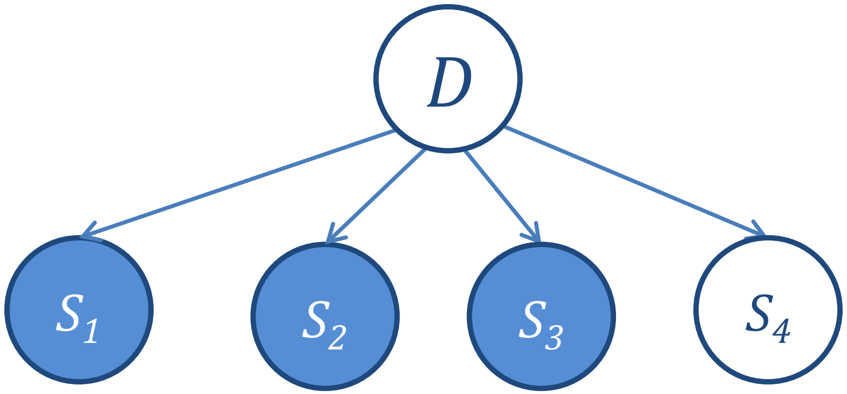
Case 1: S_1 observed to be –
SDP is 1.0



Case 2: S_1 observed to be +
SDP is 0.81

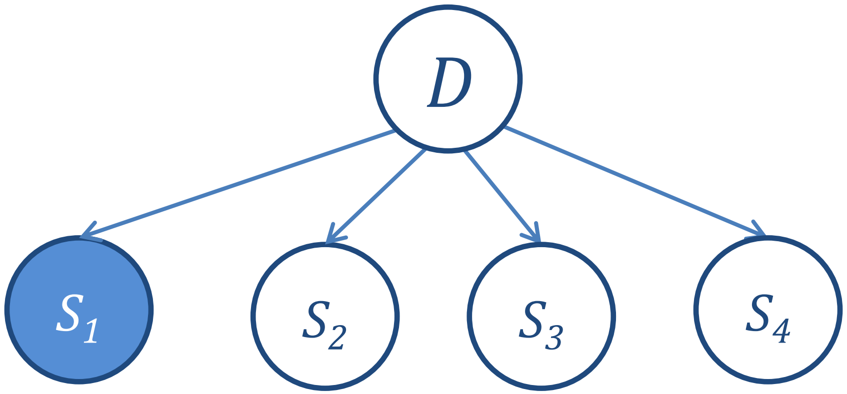
Expected SDP of observing S_1: **0.905**

SDP – Selection Criteria Example (2)



Another selection criteria has selected
several variables to observe

SDP – Selection Criteria Example (2)



We can use SDP to show that observing only a subset of these variables is necessary.

Summary

- **Same-decision probability**: useful as a tool to aid decision making.
- **Stopping criteria**: Provides a measure of how ready we are to stop making observations.
- **Selection criteria**: Helps us to select observations for a more robust decision.
- **Complexity result (see poster):** Calculating expectations (including non-myopic VOI) in a graphical model is in the same complexity class as calculating SDP.

Complexity Results

- SDP was shown to be a PP^{PP}-complete problem (Choi, Xue, Darwiche ‘12).
- PP^{PP} class – a counting variant of the class NP^{PP}.
- General problem of computing expectations **(D-EPT)** of the form is PP^{PP} -complete as well:

$$\mathrm{E} = \Sigma_{\mathbf{h}} R(\Pr(D \mid \mathbf{e}))\Pr(\mathbf{h}|\mathbf{e}) > N?$$

  – Includes SDP

  – Includes non-myopic VOI

Complexity Proof

Prove that **D-EPT** is $\mathrm{PP}^{\mathrm{PP}}$-hard:

- Reduction from the decision problem **D-SDP**.
- **D-SDP:** *Given a decision based on probability* $Pr(d\,|\,\mathbf{e})$ surpassing a threshold T, a set of unobserved variables **H**, and a probability p, is the same-decision probability:

$$\Sigma_{\mathbf{h}}\,[\Pr(d \mid \mathbf{h},\mathbf{e}) \geq T\,]\,\Pr(\mathbf{h}|\mathbf{e})$$

greater than p?

Reduction is simple – can easily define function *R* that imitates the SDP indicator function.

Complexity Proof (2)

Prove that **D-EPT** is a member of the class PP^{PP}

We provide a probabilistic polynomial-time algorithm, with access to a PP oracle, that answers **D-EPT** with probability greater than ½.

1. Sample a complete instantiation **x** from the Bayesian network, with probability *Pr(***x***)*.
2. If **x** is compatible with **e**, we can use a PP-oracle to compute t = R(*Pr*(*D* | **h,e**)).
3. Define a function $a(t) = ½ + ½ \frac{t-N}{u-l}$
4. Declare $\mathrm{E} > N$ with probability *a(t)* if **x** is compatible with **e,** ½ if **x** is not compatible with **e**.

Complexity Proof (3)

The probability of declaring $\mathrm{E} > N$ is then:

$$r = \Sigma_{\mathbf{h}}\ a(t)\ Pr(\mathbf{h,e}) + \tfrac{1}{2}\ (1 - Pr(\mathbf{e}))$$

which is greater than ½ iff:

$$\Sigma_{\mathbf{h}}\ a(t)\ Pr(\mathbf{h,e}) > Pr(\mathbf{e})/2$$

$$\Sigma_{\mathbf{h}}\ a(t)\ Pr(\mathbf{h}|\mathbf{e}) > \tfrac{1}{2}$$

$$\Sigma_{\mathbf{h}}\ (\tfrac{1}{2}\ \tfrac{t-N}{u-l})\ Pr(\mathbf{h}|\mathbf{e}) > 0$$

$$\Sigma_{\mathbf{h}}\ (t - N)\ Pr(\mathbf{h}|\mathbf{e}) > 0$$

$$\Sigma_{\mathbf{h}} R(\Pr(D \mid \mathbf{e}))\ \Pr(\mathbf{h}|\mathbf{e}) > N$$

thus *r* > ½ *iff* $\mathrm{E} > N$.