Qualitative Chain Graphs and their Use in Medicine

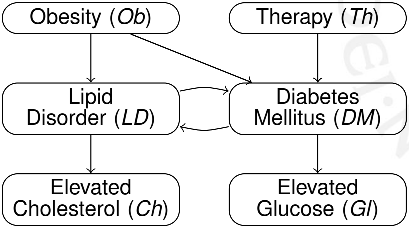
Martijn Lappenschaar, **Arjen Hommersom**, *Peter Lucas*

Department of Model-Based System Development
Institute for Computing and Information Sciences
University of Nijmegen

September 19, 2012

Motivation: modelling PGMs in medicine

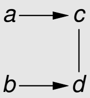
- Underlying physiological processes: dynamic (feedback) systems
  - homeostasis is ensured (*equillibrium* state)
- Disturbances may lead to suboptimal equillibria (disease)
- Treatments may affect the ‘setpoint’ of these systems
- Example:



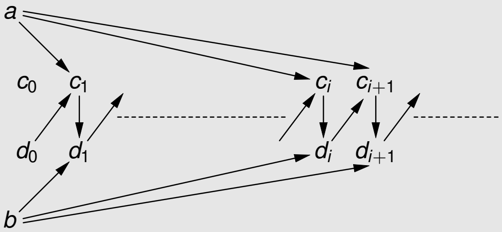
Chain graph as equillibrium of causal feedback

Example LWF chain graph (Lauritzen and Richardson)

The distribution of the chain graph model:

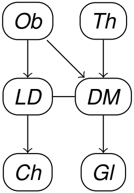


represents the equillibrium of a process represented by an infinite DAG:



Example chain graph

The example is modelled as a chain graph:



with a faithful distribution that factorises as:

$$P(Ob, Th, LD, DM, Ch, Gl) \propto P(Ch \mid LD) \cdot P(Gl \mid DM) \cdot \varphi_1(LD, DM, Ob) \cdot \varphi_2(Ob, Th, DM) \cdot P(Ob) \cdot P(Th)$$

φ_i are **black-box** parameters

Outline

- Problem: it can be difficult to exploit human knowledge in assessing chain graph parameters
- Goal: qualitative abstraction of chain graphs
- Approach: qualitative relationships based on qualitative probabilistic networks
- Qualitative and quantitative knowledge is combined
- Use such qualitative knowledge for making decisions

Qualitative probabilistic networks (QPNs)

- Qualitative abstractions of Bayesian networks
- Instead of a conditional probability $P(B \mid \pi(B))$, qualitative properties of the conditional probability are associated to each node B
  - Qualitative influences $S^{\delta}(A, B)$: the effect of a cause A on B (all other things being equal)
  - Qualitative synergies: interaction of two causes on the effect
    - Additive synergy $Y^{\delta}(\{A_1, A_2\}, B)$
    - Product synergy $X^{\delta}(\{A_1, A_2\}, b)$
- Probabilistic relationships have signs $\delta \in \{+, -, 0, ?\}$

Qualitive influences in chain graphs

- In QPNs: the influence of A on B is δ if

$$\delta = \text{sign}(P(b \mid a, x) - P(b \mid \overline{a}, x))$$

  for all configuration x of *other* parents of B; $\delta = ?$ otherwise
- Probabilistic chain graphs: neighbours need to be considered

Causal definition of influence

The influence of A on B in a context $c \in V - AB$ is

$$P(b \mid\mid a, c) - P(b \mid\mid \overline{a}, c)$$

where $P(X \mid\mid Y = y)$ denotes the probability of X after the intervention $Y = y$

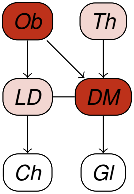
Qualitative influences in chain graphs (2)

Chain graph influence

Given two nodes A and B and a context c, then the influence of A on B in context c equals:

$$P(b \mid a, z) - P(b \mid \overline{a}, z)$$

where $c = z \cup x$, $Z = \text{bd}(B) - A$, and $X = V - ZAB$.



The influence of *Ob* on *DM* is:

$$P(dm \mid ob, Th, LD) - P(dm \mid \overline{ob}, Th, LD)$$

in any context $\{Th, LD, Ch, Gl\}$

Definition of qualitative chain graphs

QPN concepts can then be defined for qualitative chain graphs:

Influences

For example: $S^+(A, B)$ if $A \in \text{bd}(B)$ and

$$P(b \mid a, \text{bd}(B) - A) \geq P(b \mid \bar{a}, \text{bd}(B) - A)$$

Synergies

For example: $Y^+(\{A_1, A_2\}, B)$ if $A_1, A_2 \in \text{bd}(B)$, $Z = \text{bd}(B) - A_1 A_2$, and

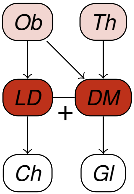
$$P(b \mid a_1, a_2, Z) - P(b \mid \bar{a}_1, a_2, Z) \geq P(b \mid a_1, \bar{a}_2, Z) - P(b \mid \bar{a}_1, \bar{a}_2, Z)$$

$\Rightarrow$ Other QPN concepts can be defined similarly

Symmetry

Theorem

It holds that qualitative signs of chain graphs are symmetric, i.e., suppose $(A, B) \in E$, then $P(b \mid a, X) - P(b \mid \bar{a}, X) \geq 0$ if and only if $P(a \mid b, Y) - P(a \mid \bar{b}, Y) \geq 0$, where $X = \text{bd}(B) - A$ and $Y = \text{bd}(A) - B$.

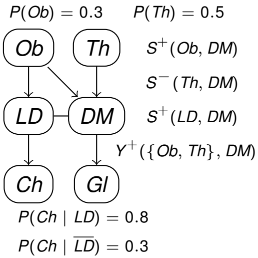


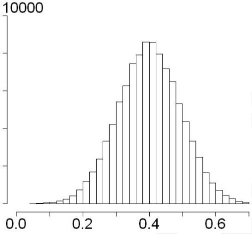
$$S^+(LD, DM) \iff S^+(DM, LD)$$

Reasoning with qualitative chain graphs

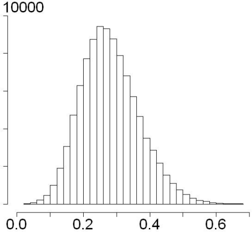
- In QPNs, conclusions are derived based on the signs (arc reversal or sign propagation)
- Alternative approach is to look upon qualitative influences/synergies as constraints (Druzdzel and van der Gaag, 1995)
  1. Sample parameters consistent with constraints
  2. Perform inference in each network
  3. Derive confidence intervals for marginals
- Can combine qualitative and quantitive information
- Locality of constraints can be exploited during sampling (come to the poster..)

Example

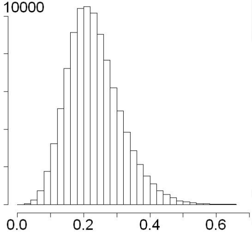




$P(Ch)$



$P(Ch \mid Th)$ (82% $> P(Ch)$)



$P(Ch \mid Th, Ob)$ (91% $> P(Ch)$)

Conclusions and future work

Conclusions:

- Feedback systems relevant in many domains (medicine, economics, embedded systems, etc)
- Qualitative chain graph models allow combining qualitative and quantitative information to model such systems
- While not precise, can be used for decision making

Future work:

- Application to multiple feedback systems (diabetes, cardiovascular domains)
- Extending the theory and efficiency of reasoning