Decision-Theoretic Troubleshooting

Hardness of Approximation

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Outline

Early Formulations of the Problem

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R. Bellman, *Dynamic Programming*, 1957.
B. Gluss, *An Optimum Policy for Detecting a Fault in a Complex System*, Operations Research, 1959.

Early Solutions – Dynamic Programming

R. Bellman - *Dynamic Programming* (1957):

> *A ball is in one of n boxes, with the a-priory probability p_k that it is in the k^{th} box. The time of examining the k^{th} box is c_k. What is the optimal policy to obtain the ball with minimum expected time?*

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The minimum expected time is

$$E(p_1, \ldots, p_n) = min_k\{c_k + (1 - p_k) \cdot E(p'_1, \ldots, p'_n)\},$$

where

$$\begin{aligned} p'_k &= 0, \text{ and} \\ p'_i &= p_i/(1 - p_k) \text{ for } i \neq k \ . \end{aligned}$$

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Solution – open the boxes according to the decreasing ratio p_k/c_k.

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- printer troubleshooting (Microsoft research),
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Bayesian networks:

- faults,
- repair actions,
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Recent dissertations:

- Håkan Warnquist (2011).
- Thorsten J. Ottosen (2012).

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You are printing a report but the colors come out very light ...

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Our goal: sequence the available repair actions so that the expected cost of the repair is as low as possible.

Troubleshooting is hard

Already very simple scenarios are *NP*-hard.

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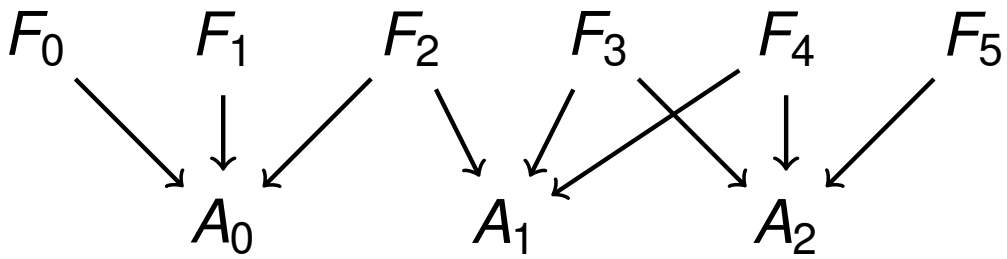
The following is an instance of the Set Cover problem:

- Let $F_0, \ldots, F_5$ be faults, all equally likely,
- let A_0, A_1, A_2 be actions, all with the same cost.



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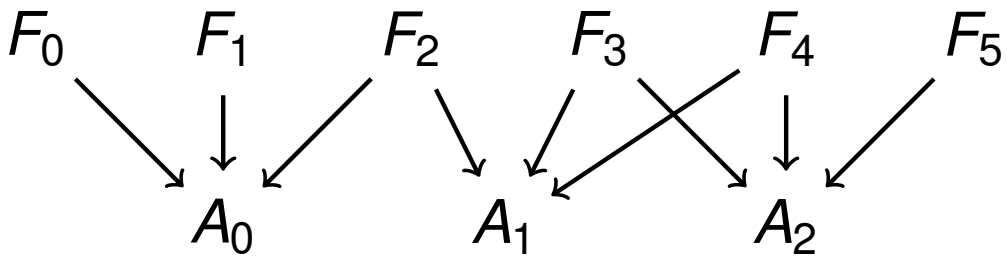

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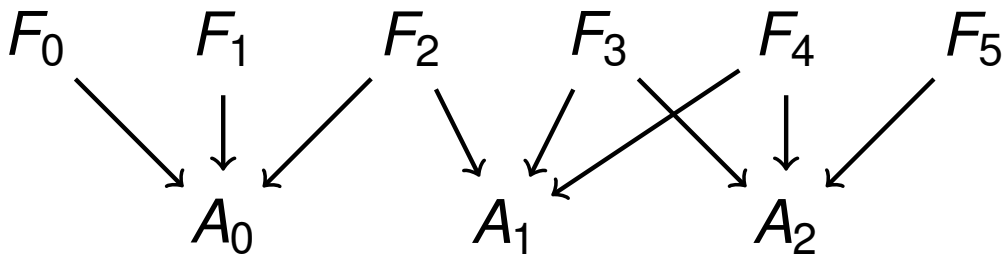

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Hardness of Approximation

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Algorithm A is a ρ-approximation algorithm ($\rho > 1$) for a minimization problem L if for all instances x of L we have

$$A(x) \leq \rho \cdot opt(x) \ ,$$

where

- $A(x)$ – the value returned by A when applied to instance x,
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Hardness of approximation result shows that for certain ρ, there is no polynomial-time ρ-approximation algorithm for L unless P=NP.

Troubleshooting with Dependent Actions

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Objective: Find a linear ordering π of actions minimizing the expected cost of repair:

$$ECR(A_{\pi(1)}, A_{\pi(2)}, \cdots) = \sum_i c(A_{\pi(i)}) \cdot \underbrace{P\Big(\bigwedge_{j<i} \{A_{\pi(j)} = \text{no}\}\Big)}_{\text{probability of reaching } A_{\pi(i)}}$$

Troubleshooting with Dependent Actions: Hardness of Approximation

Theorem

Unless P=NP, *troubleshooting with Dependent Actions has no polynomial-time* $(4-\epsilon)$*-approximation algorithm for any* $\epsilon > 0$.

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Proof by reduction from Min-sum Set Cover.

- Feige, Lovász, Tetali, *Approximating Min-sum Set Cover*, Algorithmica, 2004.

Other results

Come see the poster!

Hardness of approximation for . . .

Other results

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Hardness of approximation for . . .

- troubleshooting with multiple dependent faults,

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Hardness of approximation for ...

- troubleshooting with multiple dependent faults,
- troubleshooting with cost clusters forming a DAG.

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In the paper, we prove three new hardness of approximation results.

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Outlook:

- Greedy algorithms
  - empirical evaluation,
  - performance guarantees for special cases,
  - worst case conditions – when do the algorithms perform badly?

Thank you

Q & A